

Differentiating Inverse Hyperbolic Functions

Example

Differentiate $y = \sinh^{-1} x$

Example

Integrate $\int \frac{1}{\sqrt{a^2 + x^2}} dx$

Example

Differentiate $y = \cosh^{-1} x$

Example

Integrate $\int \frac{1}{\sqrt{x^2 - a^2}} dx$

Example

Differentiate $y = \tanh^{-1} x$

Example

Integrate $\int \frac{1}{a^2 - x^2} dx$

Example

Evaluate $\int_0^4 \frac{1}{\sqrt{9+x^2}} dx$

Example

Integrate $\int \frac{1}{\sqrt{x^2+x}} dx$

Example

Find $\int \frac{1}{7 \sinh x + 3 \cosh x - 9} dx$

Tip (Useful Substitutions)

Some helpful substitutions for integrating tougher integrals:

Expression	Substitution
$a^2 + x^2$	$x = a \tan u$ or $x = a \sinh u$
$a^2 - x^2$	$x = a \sin u$
$x^2 - a^2$	$x = a \cosh u$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \sinh^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \sqrt{a^2 + x^2} dx = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \left(\frac{x}{a} \right) + C$$

Tip (Useful Integration by parts)

Integrate by parts $\frac{du}{dx} = 1, v = f(x)$ to get $x \times f'(x)$ which we can integrate

$$\int \ln x dx = [x \ln x] - \int \left(x \cdot \frac{1}{x} \right) dx$$

$$\int \sin^{-1} x dx = [x \sin^{-1} x] - \int \left(x \cdot \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$\int \cos^{-1} x dx = [x \cos^{-1} x] - \int \left(x \cdot \frac{-1}{\sqrt{1-x^2}} \right) dx$$

$$\int \tan^{-1} x dx = [x \tan^{-1} x] - \int \left(x \cdot \frac{1}{1+x^2} \right) dx$$

$$\int \sinh^{-1} x dx = [x \sinh^{-1} x] - \int \left(x \cdot \frac{1}{\sqrt{1+x^2}} \right) dx$$

$$\int \cosh^{-1} x dx = [x \cosh^{-1} x] - \int \left(x \cdot \frac{1}{\sqrt{x^2-1}} \right) dx$$

$$\int \tanh^{-1} x dx = [x \tanh^{-1} x] - \int \left(x \cdot \frac{1}{1-x^2} \right) dx$$

Tip (Integration by Parts to Remove a function)

$$\int x \cos x \, dx = [x \cos x] - \int 1 \cdot \sin x \, dx$$
$$\int x^n \cos x \, dx = [x^n \cos x] - \int nx^{n-1} \sin x \, dx$$
$$\int xe^x \, dx = [xe^x] - \int 1 \cdot e^x \, dx$$

Tip

Things you should always think about when integrating:

- Can I use partial fractions?
- Does the numerator look like the derivative of the denominator? [$\ln(\cdot)$]
- Can I complete the square?
- Does it look like one of the substitutions above?
- Can I integrate by parts?